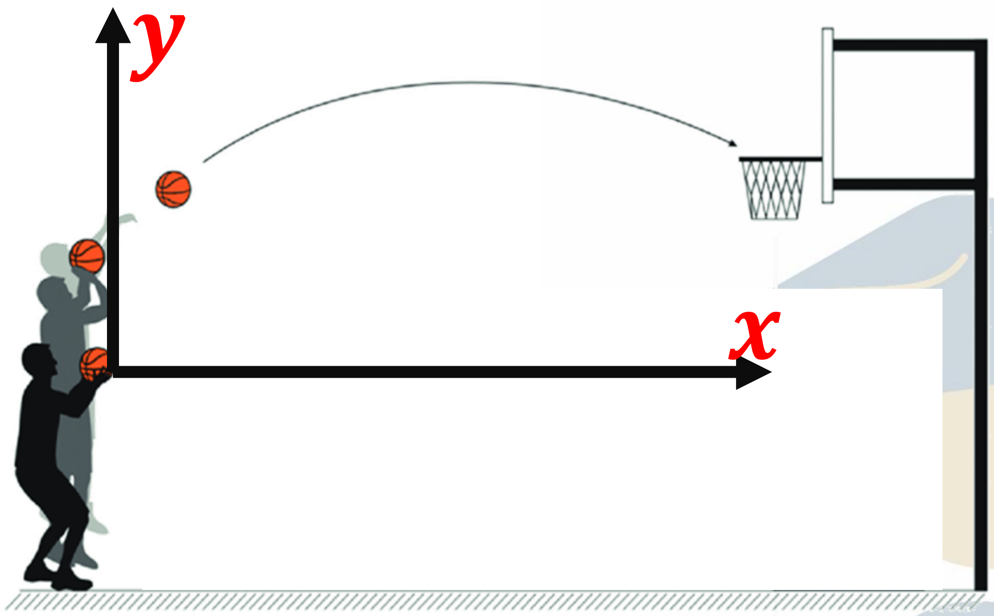


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif


Quiz Time

Quiz 1: (10 pts) 30min

Angry bird "Chuck" represented here by point C is launched from point "A" at time $t_0 = 0$ to hit a "pig" represented by point "P" as shown in the following figure.

The position vector of (C), in the given (x O y) reference, is:

$$\vec{r}\begin{Bmatrix} x = 2t \\ y = -t^2 + 3t + 1 \end{Bmatrix}$$ where t in s and r in m

Quiz 1: (10 pts) 30 min

1. Determine the time expressions of the velocity vector and its value of (C).
2. Determine the expressions of the acceleration vectors and its value of (C).
3. Determine the launch speed and angle of “Chuck”.
4. Determine the coordinates of the highest point ‘B’ reached by (C).
5. Determine the tangential acceleration vector on the highest point ‘B’.

1.Determine the time expressions of the velocity vector and its value of (C).

$$\vec{r} = \begin{cases} x = 2t \\ y = -t^2 + 3t + 1 \end{cases}$$

$$\vec{r} = x\,\vec{i} + y\,\vec{j}$$

$$\vec{r} = 2t.\vec{i} + \left(-t^2 + 3t + 1\right)\vec{j}$$

$$\vec{v} = \vec{r}\,'$$

$$\vec{v} = 2.\vec{i} + (-2t + 3).\vec{j}$$

$$\vec{v} = 2.\vec{i} + (-2t + 3).\vec{j}$$

$$V = \sqrt{V_x^2 + V_y^2}$$

$$V = \sqrt{(2)^2 + (-2t + 3)^2}$$

$$V = \sqrt{4 + 4t^2 - 12t + 9}$$

$$V = \sqrt{4t^2 - 12t + 13}$$

$$\vec{V} = 2.\vec{i} + (-2t + 3).\vec{j}$$

2.Determine the expressions of the acceleration vectors and its value of (C

$$\vec{a} = \vec{V}\,'$$

$$\vec{a} = 0.\vec{i} - 2.\vec{j}$$

$$\vec{a} = -2.\vec{j}$$

$$a = \sqrt{a_x^2 + a_y^2}$$

$$a = \sqrt{(0)^2 + (2)^2}$$

$$a = \sqrt{0 + 4}$$

$$a = 2m / s^2$$

$\vec{V} = 2.\vec{i} + (-2t + 3).\vec{j}; \vec{a} = -2.\vec{j}$

3. **Determine the launch speed and angle of "Chuck"**

At t=0:

$$\overrightarrow{v_0} = 2\,\vec{i} + (-2 \times 0 + 3)\vec{j}$$

$$\overrightarrow{v_0} = 2\,\vec{i} + 3\,\vec{j}$$

$$\overrightarrow{v_0} = 2\,\vec{i} + 3\,\vec{j}$$

$$V_0 = \sqrt{V_{0x}^2 + V_{0y}^2}$$

$$V_0 = \sqrt{2^2 + 3^2}$$

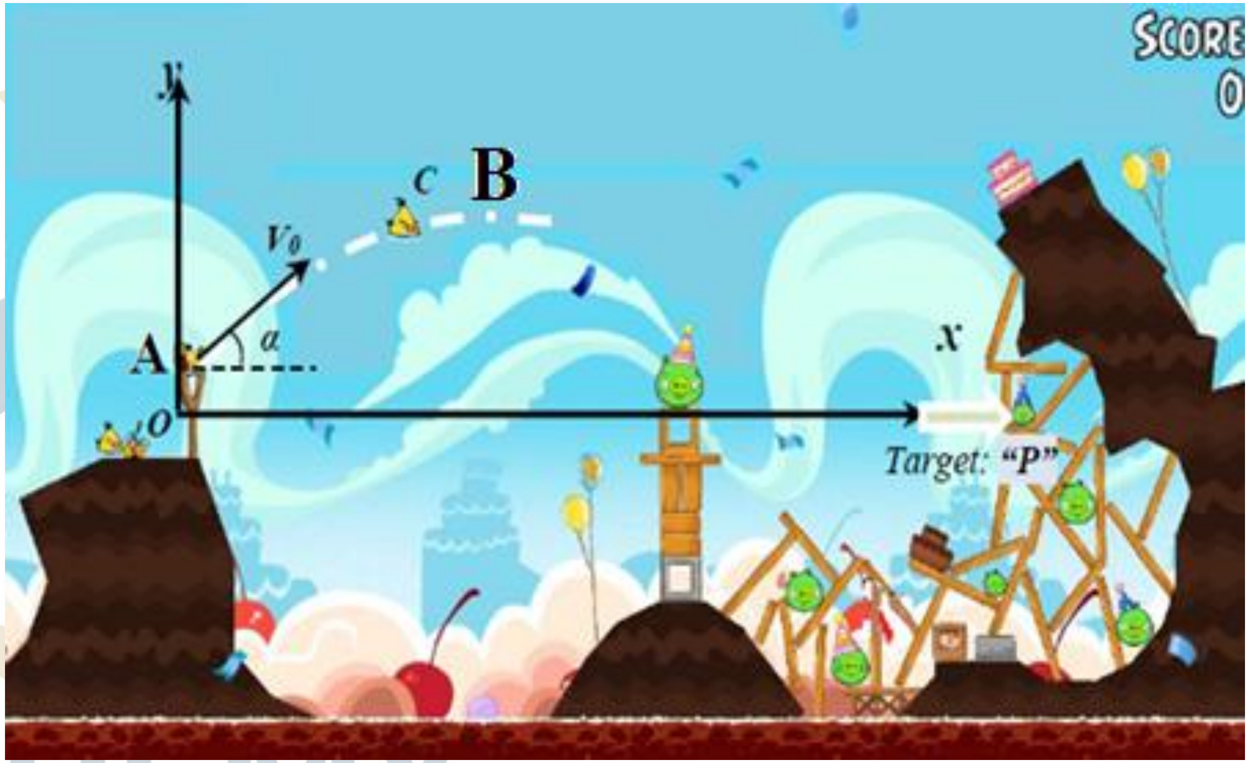
$$V_0 = 3.6 m/s$$

$$tan\alpha = \frac{3}{2} = 1.5$$

$$\tan\alpha = \frac{opp}{adj} = \frac{v_{0y}}{v_{0x}}$$

$$\alpha = \tan^{-1}(1.5) = 56°$$

4. Determine the coordinates of the highest point 'B' reached by (C).



At B: $v_y = 0$

$$-2t + 3 = 0$$

$$3 = 2t \Rightarrow t = \frac{3}{2} = 1.5\ s$$

$$\vec{r} = \begin{cases} x = 2t \\ y = -t^2 + 3t + 1 \end{cases}$$

$$x_B = 2 \times 1.5 = 3m$$

$$y_B = -(1.5)^2 + 3 \times 1.5 + 1$$

$$y_B = 3.25m$$

$\vec{v} = 2.\vec{i} + (-2t + 3).\vec{j}\ ;\ V = \sqrt{4t^2 - 12t + 13}$

5. Determine the tangential acceleration vector on the highest point ‘B’.

$$a_t = v'$$

$$a_t = \frac{8t - 12}{2\sqrt{4t^2 - 12t + 13}}$$

$$a_t = \frac{4t - 6}{\sqrt{4t^2 - 12t + 13}}$$

$$\vec{v} = v.\vec{u_t} \Rightarrow \vec{u_t} = \frac{\vec{v}}{v}$$

$$\vec{u_t} = \frac{2\,\vec{i} + (-2t + 3)\vec{j}}{\sqrt{4t^2 - 12t + 13}}$$

$$a_t = \frac{4t-6}{\sqrt{4t^2-12t+13}}$$

On B; t = 1.5 s

$$a_t = \frac{4(1.5)-6}{\sqrt{4(1.5)^2-12(1.5)+13}}$$

$$a_t = \frac{0}{\sqrt{13}}$$

$$a_t = 0$$

$$\overrightarrow{u_t} = \frac{2\,\vec{i} + (-2t+3)\vec{j}}{\sqrt{4t^2-12t+13}}$$

$$\overrightarrow{a_t} = a_t \times \overrightarrow{u_t}$$

$$\overrightarrow{a_t} = (0) \times \overrightarrow{u_t}$$

$$\overrightarrow{a_t} = \vec{0}$$

6. **Deduce the characteristics of the normal acceleration vector on the highest point ‘B’.**
7. **Determine the equation of the trajectory followed by (C).**
8. **Knowing that “P” is located at (9m; – 0.5m); would (C) hit its target point? Justify**

$\overrightarrow{a_t} = \vec{0}; \vec{a} = 2.\vec{j}$

Be Smart ACADEMY

6.Deduce the characteristics of the normal acceleration vector on the highest point ‘B’.

$$\vec{a} = \overrightarrow{a_t} + \overrightarrow{a_n}$$

$$\overrightarrow{a_n} = \vec{a} - \overrightarrow{a_t}$$

$$\overrightarrow{a_n} = -2\,\vec{j} - \vec{0}$$

$$\overrightarrow{a_n} = -2\,\vec{j}$$

- Origin: point ‘B’
- Direction: down
- Line of action: vertical
- Magnitude: $2\ m\ /\ s^2$

7. **Determine the equation of the trajectory followed by (C).**

$$\vec{r} = \begin{cases} x = 2t \\ y = -t^2 + 3t + 1 \end{cases}$$

$x = 2t$ ⇒ $t = \frac{x}{2}$

Substitute x in the expression of y

$$y = -t^2 + 3t + 1$$

$$y = -\left(\frac{x}{2}\right)^2 + 3\left(\frac{x}{2}\right) + 1$$

$$y = -\frac{x^2}{4} + \frac{3}{2}x + 1$$

$$y = -0.25x^2 + 1.5x + 1$$

Parabolic shape

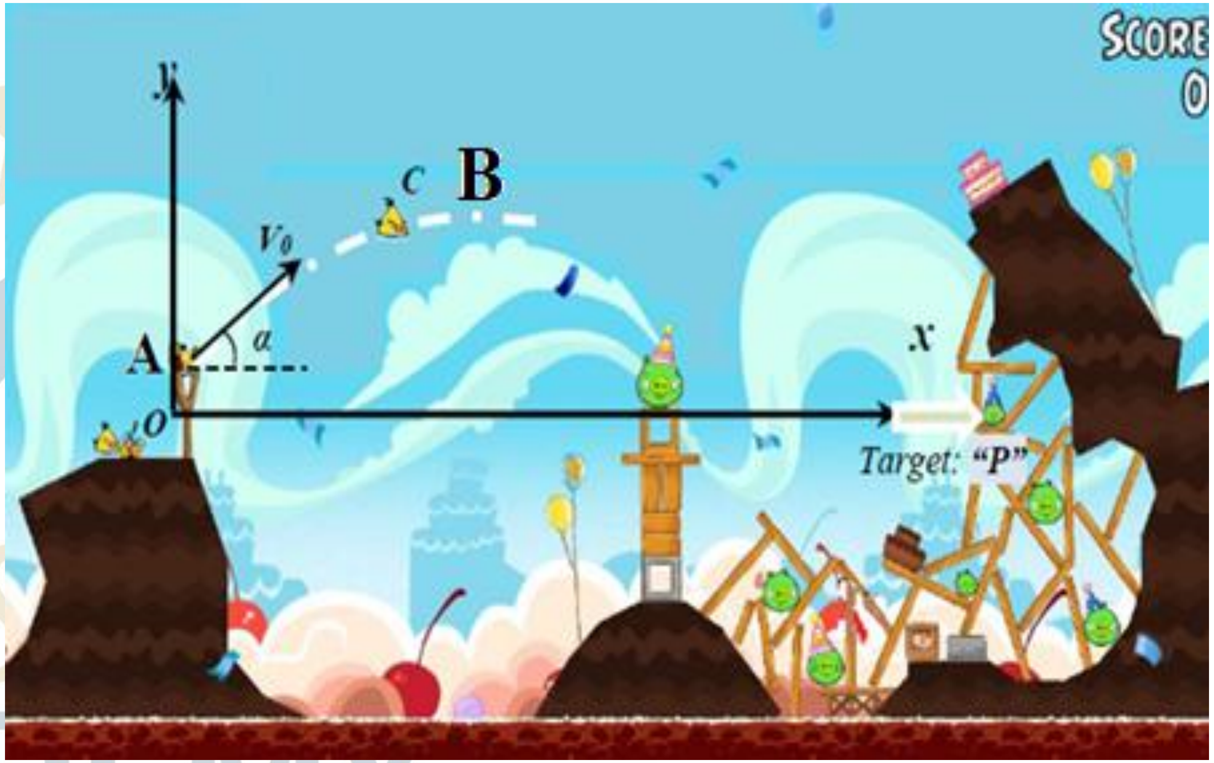
8.Knowing that "P" is located at (9m; – 0.5m); would (C) hit its target point? Justify.

$$y = -0.25x^2 + 1.5x + 1$$

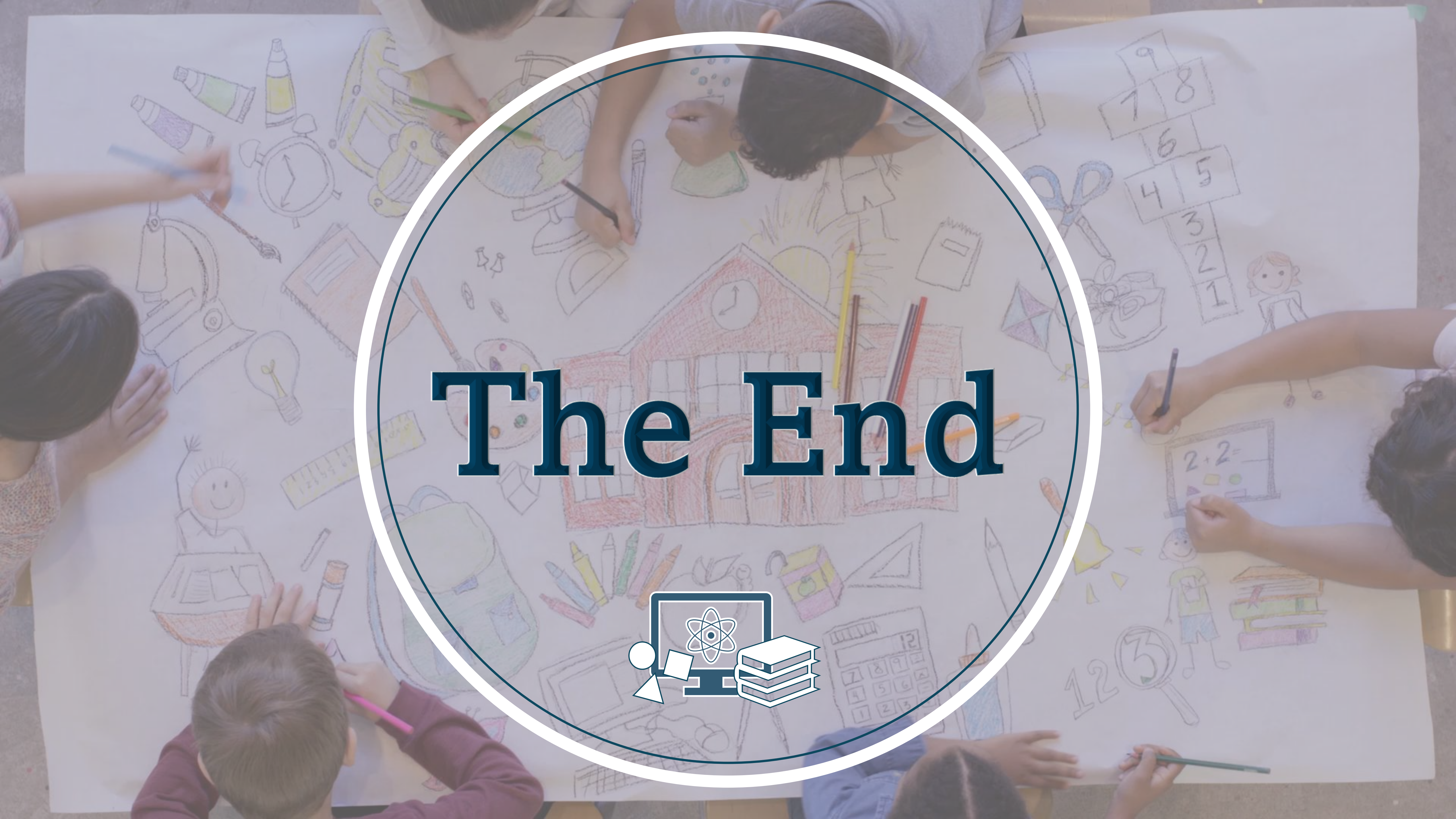
$$-0.5 = -0.25 \times 9^2 + 1.5 \times 9 + 1$$

$$-0.5 = -20.25 + 13.5 + 1$$

$$-0.5 = -5.75 \quad \textit{impossible}$$



The point "P" doesn't exist on the trajectory, Then (C) doesn't hit its target point "P"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Quiz 2: **(10 pts)** **20 min**

The motion of a particle is defined by:

$$\vec{r} = 2sin(3t).\vec{\imath} + 2cos(3t).\vec{\jmath}$$ **, where [r and t in SI]**

1.Determine the speed of the particle at any time t.

2.What is the shape of its trajectory?

3.Deduce that the motion is U.C.M.

4.Locate the particle at the instants t = 0 s and t = $\frac{\pi}{6}$ s.

5.Determine the average velocity vector between the above instants.

6.Find a relation between the position and the acceleration vectors.

$$\vec{r} = 2sin(3t).\vec{i} + 2cos(3t).\vec{j}$$

1. Determine the speed of the particle at any time t.

The velocity vector is the derivative of position vector:

$$\vec{v} = \vec{r}\,'$$

$$\vec{V} = [2 \times 3cos(3t)].\vec{i} - [2 \times 3\sin(3t)].\vec{j}$$

$$\vec{V} = 6cos(3t).\vec{i} - 6\,sin(3t)\,.\vec{j}$$

$$\vec{r} = 2sin(3t).\vec{i} + 2cos(3t).\vec{j}; \vec{V} = 6cos(3t).\vec{i} - 6\,sin(3t)\,.\vec{j}$$

$$\sqrt{V_x^2 + V_y^2}$$

$$v = \sqrt{(6\cos(3t))^2 + (6\sin(3t))^2}$$

$$v = \sqrt{36cos^2(3t) + 36sin^2(3t)}$$

$$v = \sqrt{36[cos^2(3t) + sin^2(3t)]}$$

$$\cos^2(\alpha) + \sin^2(\alpha) = 1$$

$$v = \sqrt{36(1)}$$

$$v = 6m/s$$

2. What is the shape of its trajectory?

$$x = 2\sin(3t)$$

$$\sin(3t) = \frac{x}{2} \Rightarrow \sin^2(3t) = \frac{x^2}{4}$$

$$y = 2\cos(3t)$$

$$\cos(3t) = \frac{y}{2} \Rightarrow \cos^2(3t) = \frac{y^2}{4}$$

Add the two equations:

$$\cos^2(3t) + \sin^2(3t) = \frac{x^2}{4} + \frac{y^2}{4}$$

$$\cos^2(3t) + \sin^2(3t) = 1$$

$$1 = \frac{x^2}{4} + \frac{y^2}{4}$$

$$x^2 + y^2 - 2ax - 2by + c = 0$$

Circular shape

3. Deduce that the motion is U.C.M.

Since the speed is constant at $v = 6\ m/s$ Uniform motion

Since the trajectory is circular shape then: Circular motion

Then the motion is **U.C.M**

$$\vec{r} = 2sin(3t).\vec{i} + 2cos(3t).\vec{j}$$

Be Smart ACADEMY

4. Locate the particle at the instants t = 0 s and t = $\frac{\pi}{6}$ s.

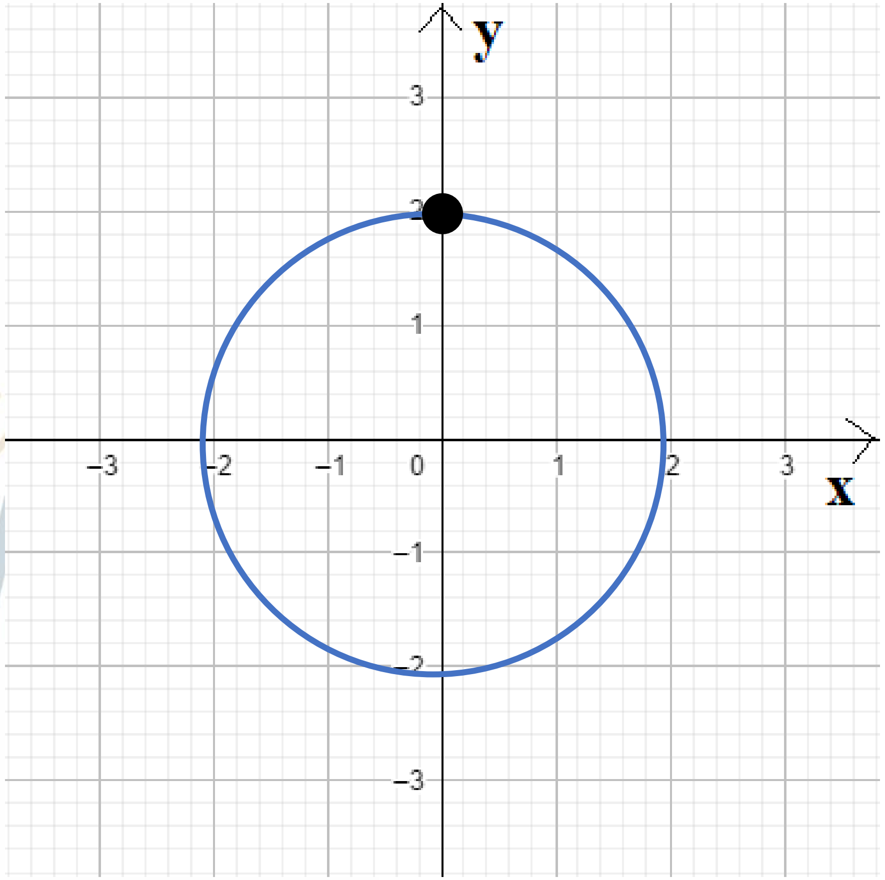
For t = 0:

$x = 2sin(3t) \Rightarrow x = 2sin(3 \times 0)$

$x = 2sin(0) = 0$

$y = 2cos(3t) \Rightarrow y = 2cos(0)$

$y = 2$



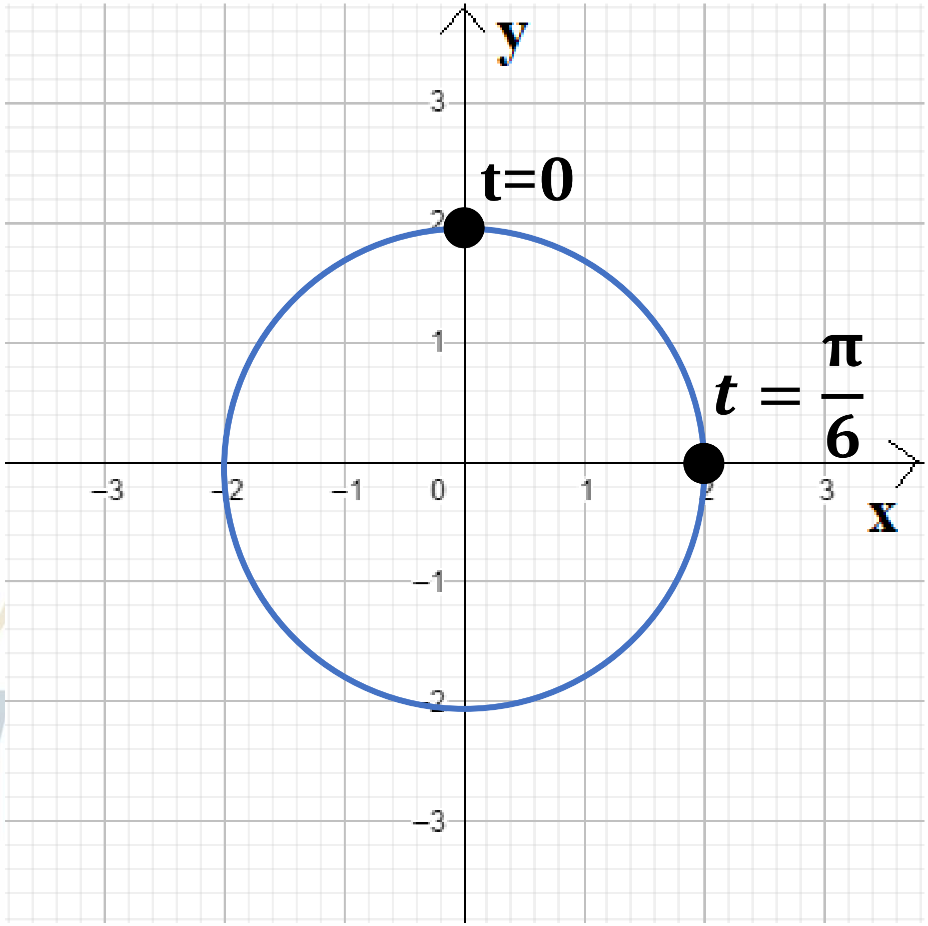
The particle is at point (0;2)

For $t = \frac{\pi}{6} s$:

$$x = 2sin(3 \times \frac{\pi}{6}) = 2sin(\frac{\pi}{2}) = 2$$

$$y = 2\cos(3 \times \frac{\pi}{6}) = 2\cos(\frac{\pi}{2}) = 0$$

The particle is at point (2;0)



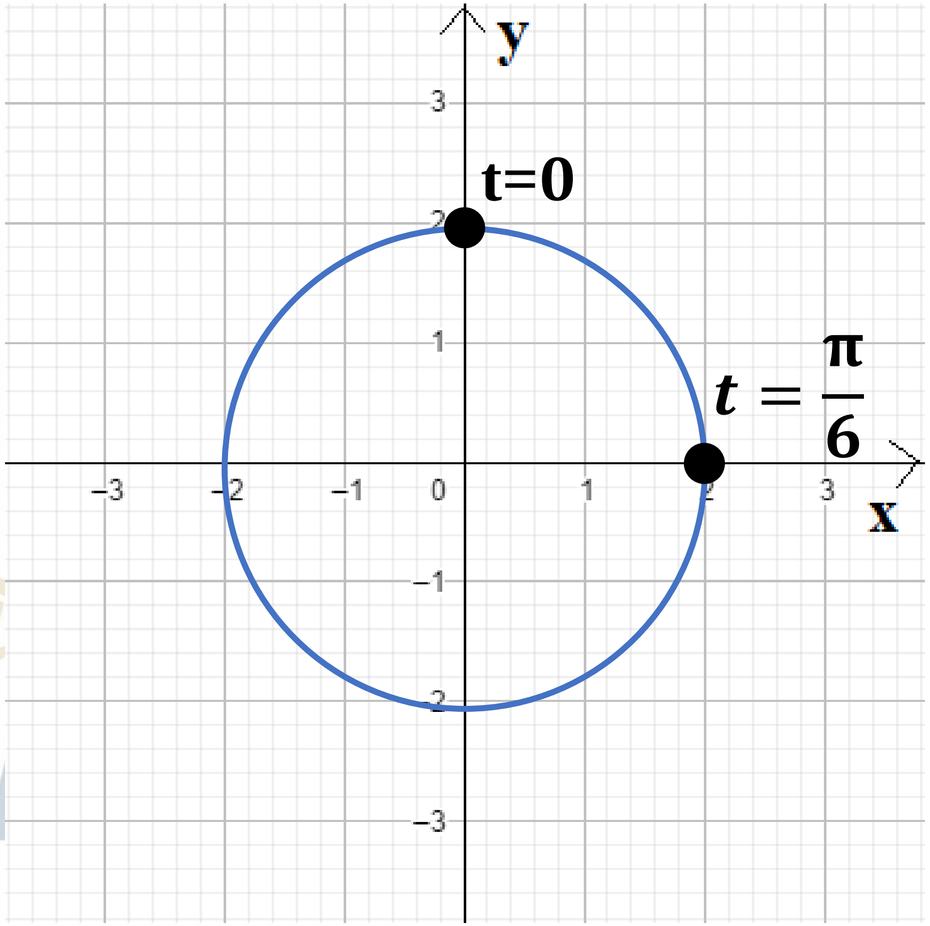
5. **Determine the average velocity vector between the above instants**

$$\overrightarrow{v_{av}} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\overrightarrow{r_2} - \overrightarrow{r_1}}{t_2 - t_1}$$

$$\overrightarrow{r_1} = \overrightarrow{r_{(t=0)}} = 0\,\vec{i} + 2\,\vec{j}$$

$$\overrightarrow{r_2} = \overrightarrow{r_{(t=\frac{\pi}{6})}} = 2\,\vec{i} + 0\,\vec{j}$$

$$\overrightarrow{v_{av}} = \frac{\overrightarrow{r_2} - \overrightarrow{r_1}}{t_2 - t_1} = \frac{2\,\vec{i} - 2\,\vec{j}}{\frac{\pi}{6} - 0}$$



$$\overrightarrow{v_{av}} = 3.81\,\vec{i} - 3.81\,\vec{j}$$

$\vec{r} = 2sin(3t).\vec{i} + 2cos(3t).\vec{j}; \vec{V} = 6cos(3t).\vec{i} - 6\,sin(3t)\,.\vec{j}$

6.Find a relation between the position and the acceleration vectors.

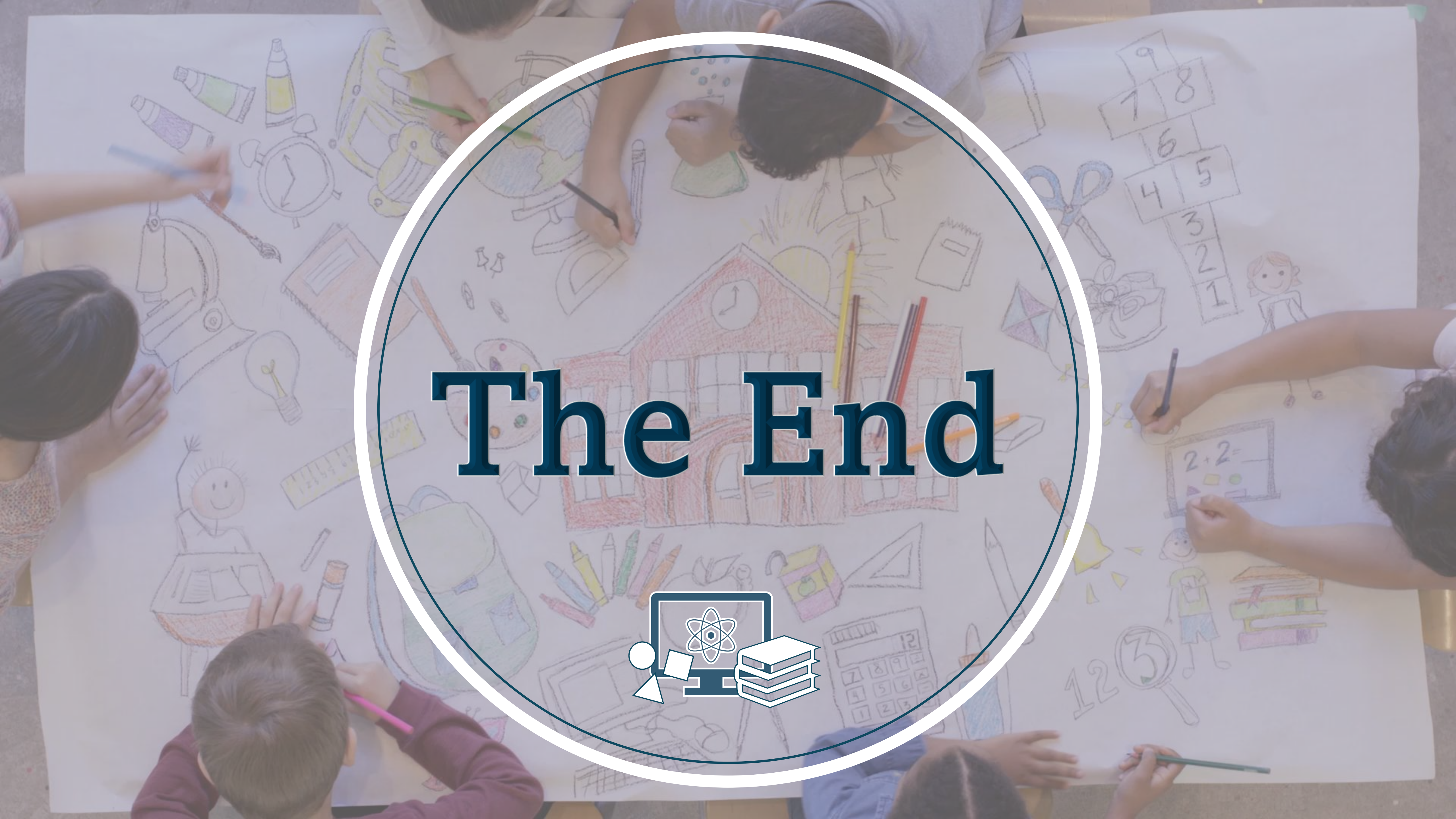
$$\vec{a} = \vec{v}\,'$$

$$\vec{a} = 6 \times 3sin(3t).\vec{i} - 6 \times 3sin(3t).\vec{j}$$

$$\vec{a} = -\,18\,sin(3t)\,\vec{i} - 18\,cos(3t)\vec{j}$$

$$\vec{a} = -\,9[\,2\,sin(3t)\,\vec{i} + 2\,cos(3t)\vec{j}]$$

$$\vec{a} = -9\,\vec{r}$$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